

## FINITE ELEMENT ANALYSIS OF MICROSTRIP LINE ON FERROELECTRIC (Ba - Sr)TiO<sub>3</sub> SUBSTRATE

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### Abstract

The characteristics of microstrip line on ferroelectric (Ba - Sr)TiO<sub>3</sub> substrate is analyzed. The static field analysis is performed iteratively to obtain the position-dependent dielectric constant in BST substrate with DC bias voltage. On the assumption of small AC field compared to DC field, the propagation constant and the characteristic impedance of the microstrip line on such a biased BST substrate is found by edge-based finite element method. The results can be used for the accurate design of various devices using BST such as dielectric phase shifter, voltage controlled resonators, etc.

### I. Introduction

Some nonlinear dielectric materials are under research to develop microwave circuit components [1][2]. The permitivities of these materials vary with the applied DC electric field while the permeability of ferrite changes with the applied magnetic field. Among these materials, (Ba - Sr)TiO<sub>3</sub> series are suitable for application due to their very high relative permitivity and significant change in relative permitivity with the DC biasing electric field at microwave frequencies [3][4].

It is worthwhile to analyze the propagation characteristics of nonlinear dielectric, especially, (Ba - Sr)TiO<sub>3</sub>, transmission line for further application.

In this paper, the DC bias effect is analyzed by solving static Poisson's equation iteratively for a

given bias condition and then the propagation characteristics and the characteristic impedances of (Ba - Sr)TiO<sub>3</sub> microstrip transmission line are analyzed considering the DC bias effect. The finite element method based upon edge element resulting in no spurious solutions is used to calculate the propagation characteristics because of the inhomogeneity of the considered transmission line

### II. Properties and Modelling of (Ba - Sr)TiO<sub>3</sub> Dielectric

The dielectric properties of (Ba - Sr)TiO<sub>3</sub> series are also dependent upon its composition, frequency as well as DC biasing electric field. It was reported that the relative permitivity and the loss tangent value are 760 and 0.02 at 9.45 GHz at 20 °C for Ba<sub>0.5</sub>Sr<sub>0.5</sub>TiO<sub>3</sub>[1]. The temperature variation, dependence on frequencies should be considered for accurate analysis. But, since the objective of the present paper is to study the propagation characteristics of microwave guided along the nonlinear dielectrics with very high dielectric constant, the dielectric property of (Ba - Sr)TiO<sub>3</sub> is simply modeled as Fig.1 for the specific composition, Ba<sub>0.5</sub>Sr<sub>0.5</sub>TiO<sub>3</sub>, the most practical composition, at microwave frequencies, on the basis of the previously reported data [1][3][4]. In Fig.1, the dielectric constant of Ba<sub>0.5</sub>Sr<sub>0.5</sub>TiO<sub>3</sub> is assumed to be 760 with no biasing electric field and to change by 16% with 2 V/um biasing electric field. Also, Ba<sub>0.5</sub>Sr<sub>0.5</sub>TiO<sub>3</sub> is assumed to be lossless.

The model for Ba<sub>0.5</sub>Sr<sub>0.5</sub>TiO<sub>3</sub> microstrip line is illustrated in Fig.2. The relative permitivity in region II is a function of position after applying the bias potential.

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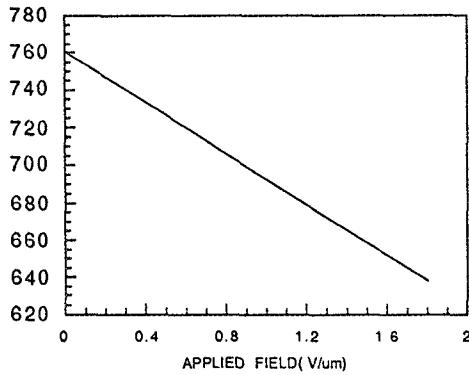


Fig.1. Change in dielectric constant with applied DC biasing fields for  $\text{Ba}_{0.5}\text{Sr}_{0.5}\text{TiO}_3$

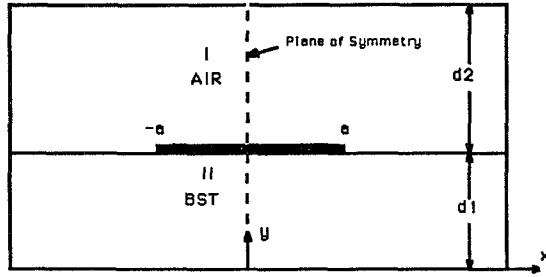


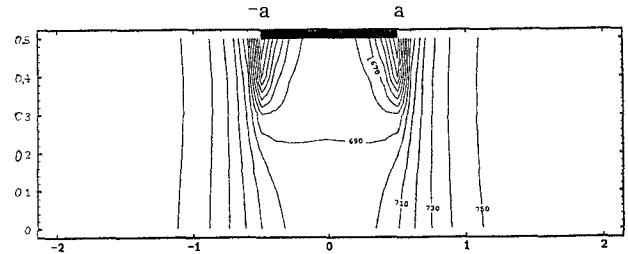
Fig.2. Modeling for Microstrip line

### III. DC Nonlinear Analysis and Results

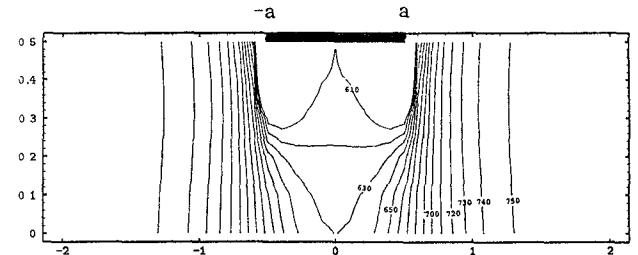
Since the amplitude of microwave signal is much small compared to that of applied DC potential in considered structure, microwave signal does not affect the permittivity of BST.

The biasing potential applied on microstrip line changes the permittivities of BST and affects the propagation characteristics. The permittivity of Region II varies with position because of different electric field intensity of position. To obtain permittivity distribution for given bias potential, the whole region II is divided into many small subregions and the permittivity of each small subregion is assumed to be constant. And, then the static Poisson's equation is solved using finite element method iteratively. First, the permittivities of all subregion is assumed to be 760, that of no bias condition. The static electric field calculated from solving the Poisson's equation modifies the permittivity of each element according to its electric field intensity and this modification changes the electric field. As the iteration number

increases, the permittivity of each element converges to a specific value. The final relative permittivity distributions for 1000V and 500V biasing potentials when  $d1=0.5$  mm and  $2a=1$  mm, are illustrated in Fig.3.



(a) when applied biasing potential is 500 V



(b) when applied biasing potential is 1000 V

Fig.3. Final relative permittivity distribution for  $d1=0.5$  mm and  $2a=1$  mm

### IV. Propagation Characteristics Analysis

#### A. Theory and Derivation of Matrix Equation

Various methods have been used to calculate the propagation characteristics of guiding structures. Finite Element method is especially suitable to present microstrip line because permittivity varies with position. The conventional  $E_z$ - $H_z$  formulation, 3-vector  $H$ -formulation with node-based element could be used. But, these formulations can give nonphysical or spurious solutions[5][6].

In this paper, formulation based upon edge element which results in no spurious solutions, is used. The triangular edge element is used for transverse electric field components and conventional node-based 1st order Lagrange interpolation function is used for z component since z direction is normal to the waveguide cross section and tangential to element boundaries. And, the microstrip is assumed to be infinitesimally thin. Also, variable transformation proposed in [7] is used to make the square of propagation constant eigenvalue of resulting eigen matrix equation.

In lossless case, the matrix equation can be derived by Rayleigh-Ritz Method or by Galerkin's Method using weighted residual.

### B. Application of the Finite Element Method

The dominant mode can be obtained by solving half of the whole structure considering the symmetry plane of Fig.2, to be a magnetic wall, which reduces memory and calculation time. Since coefficients of nodes and edges on the electric wall are zero, the rows and the columns corresponding to these edges and nodes can be deleted from matrix equation. After imposing the Dirichlet boundary condition, the reduced eigenmatrix equation can be solved by IMSL or EISPACK routines.

### V. Results

The present FEM code is applied to conventional microstrip line and excellent agreement is obtained with the available data [5][8] and no spurious solutions occur as expected.

The effective permittivities and the characteristic impedances of  $\text{Ba}_{0.5}\text{Sr}_{0.5}\text{TiO}_3$  microstrip line for  $d_1=0.5$  mm and  $2a=1$  mm, and for  $d_1=0.5$  mm and  $2a=0.6$  mm with no biasing potential, 500V and 1000V biasing potentials are shown in Fig.4 and Fig.5, respectively. The characteristic impedance increase as the width of microstrip become narrow but the characteristic impedances for both cases are less than  $10 \Omega$ . The error occurring when simply assuming the permittivity of whole region to be that calculated from approximate electric field intensity between microstrip and ground plane, is shown in Fig.6.

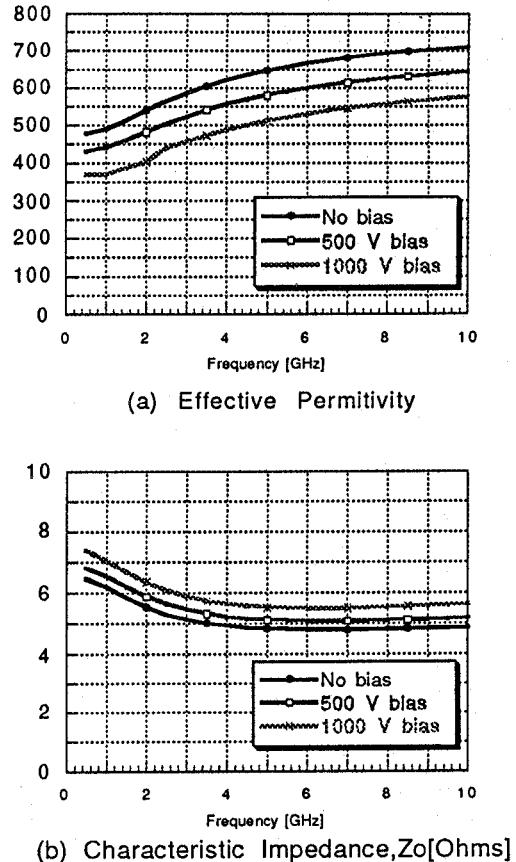
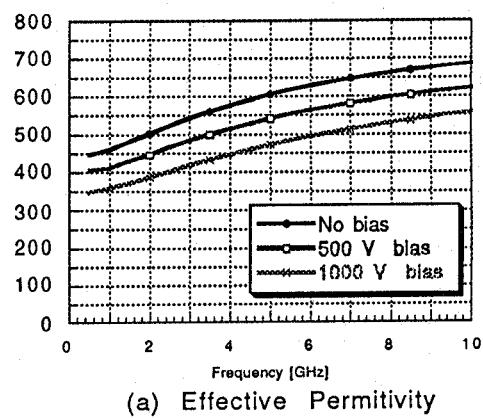


Fig.4. Propagation Characteristics when  $d_1=0.5$  mm and  $2a=1$  mm



(a) Effective Permittivity

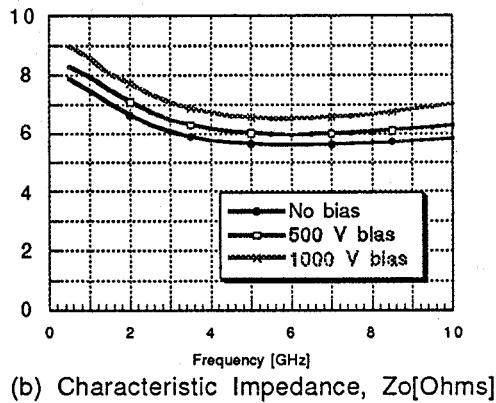


Fig.5. Propagation Characteristics when  $d_1=0.5$  mm and  $2a=0.6$  mm

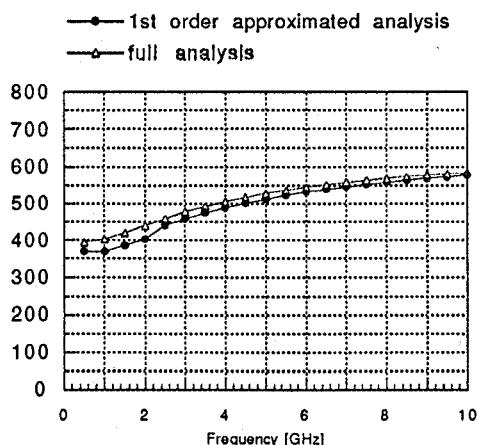


Fig.6. Effective Permittivity Difference between full analysis and approximation for 1000 V biasing potential when  $d_1=0.5$  mm and  $2a=1$  mm

## VI. Summary.

The  $(\text{Ba} - \text{Sr})\text{TiO}_3$  microstrip line can be used as a delay line for an electric-field-controlled phase shifter. In order to design the BST phase shifter, it is important to obtain the position-dependent dielectric constant of the substrate with an applied DC bias voltage since the dielectric constant of the BST substrate varies with position due to its nonlinearity. After the profile of the dielectric constant in BST substrate with given DC bias is obtained, the propagation constant and the characteristic impedance of the microstrip line on

inhomogeneous BST substrate are calculated using the edge-based FEM. The results can be used to design other voltage-controlled circuit elements such as voltage-controlled resonators, microstrip couplers, and variable capacitors.

## References

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